Government of **Western Australia**
School Curriculum and Standards Authority

ATAR course examination, 2017
Question/Answer booklet

MATHEMATICS SPECIALIST

Place one of your candidate identification labels in this box.

Ensure the label is straight and within the lines of this box.

Section Two: Calculator-assumed

Student number: In figures | | | | | | | | | |

In words _____

_____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Number of additional answer booklets used (if applicable):

Materials required/recommended for this section

To be provided by the supervisor

This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	53	35
Section Two: Calculator-assumed	11	11	100	97	65
				Total	100

Instructions to candidates

1. The rules for the conduct of the Western Australian external examinations are detailed in the *Year 12 Information Handbook 2017*. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your answers to the specific questions asked and to follow any instructions that are specific to a particular question.

4. Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

Section One: Calculator-assumed **65% (97 Marks)**

This section has **11** questions. Answer **all** questions. Write your answers in the spaces provided.

Supplementary pages for the use of planning/continuing your answer to a question have been provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

Working time: 100 minutes.

Question 9 **(6 marks)**

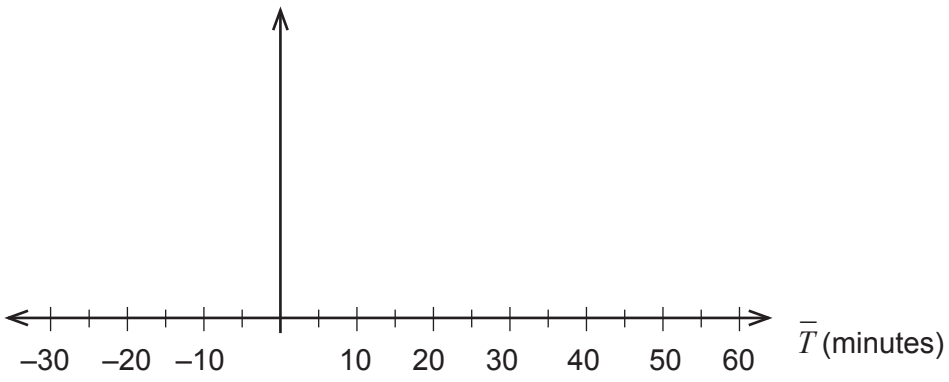
The time T in minutes that a particular flight arrives later than its scheduled time is uniformly distributed with $-30 \leq T \leq 60$. The population mean is $\mu(T) = 15$ and the population variance is $\sigma^2(T) = 675$.

A sample of 30 arrival times is taken and the sample mean $\overline{T}$ is calculated.

(a) Determine $P(10 \leq \overline{T} \leq 20)$ correct to 2 decimal places. (3 marks)

(b) If a large number of samples, each with 30 arrival times, is taken, sketch the likely distribution of the sample mean $\overline{T}$ below.

In the diagram indicate or refer to the calculation from part (a). (3 marks)



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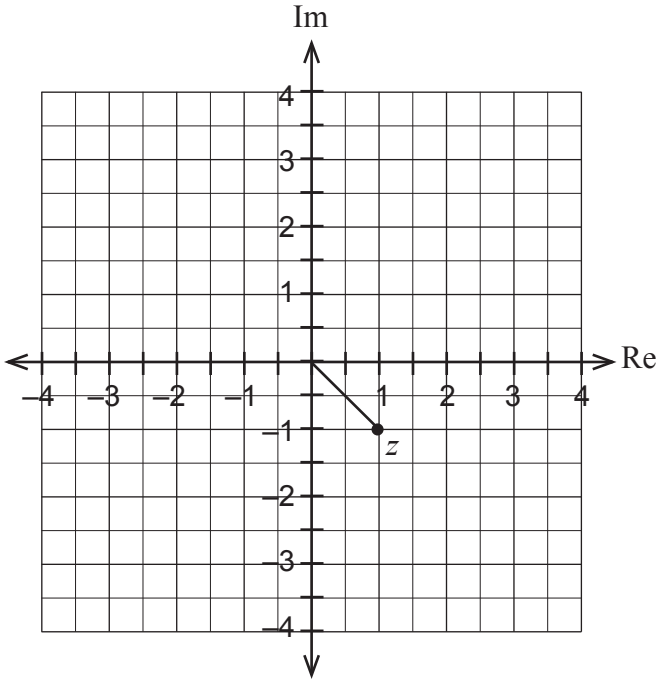
Question 10 **(7 marks)**

Consider $z = 1 - i$ shown in the complex plane below.



(a) Express z in polar form. (1 mark)

(b) Hence express z^2, z^3 and z^4 in exact polar form. (2 marks)

See next page

(c) Sketch the complex numbers z^2, z^3, z^4 as vectors in the given Argand diagram. (2 marks)

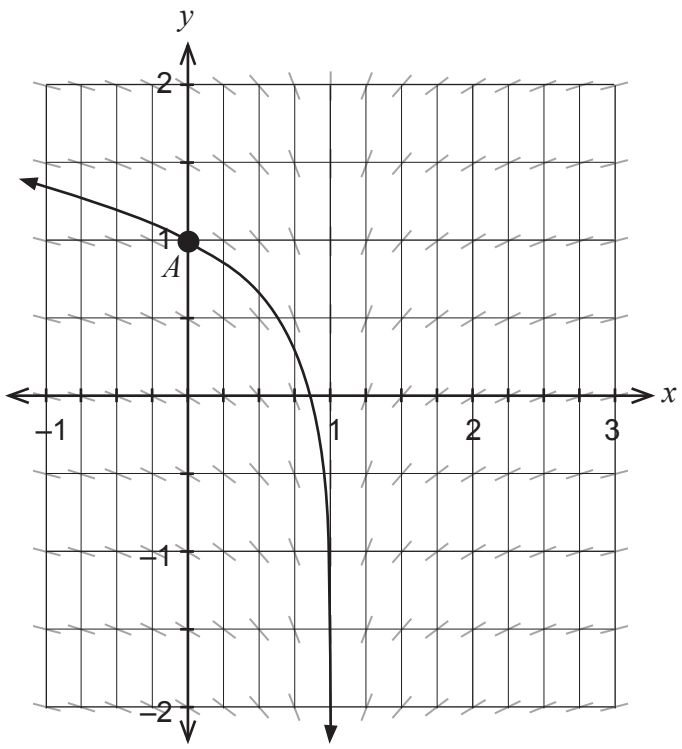
Consider the geometric transformation(s) applied to transform $z \rightarrow z^2 \rightarrow z^3 \rightarrow z^4$ etc.

(d) Describe the geometric transformation(s) performed by successive multiplication by z. (2 marks)

See next page

Question 11 **(7 marks)**

A series of magnets is placed under a glass pane and some iron filings are sprinkled onto the glass. The orientation or slope of the iron filings, as determined by the magnetic field, is shown below. One of the lines of magnetic force that passes through the point $A\ (0,1)$ is also shown.



The slope field is given by $\frac{dy}{dx} = \frac{1}{2x-2}\ ,\ x \neq 1.$

(a) Determine the value of the slope field at the point $A\ (0,1)$. (2 marks)

(b) Explain the orientation of the iron filings at $x = 1$. (1 mark)

See next page

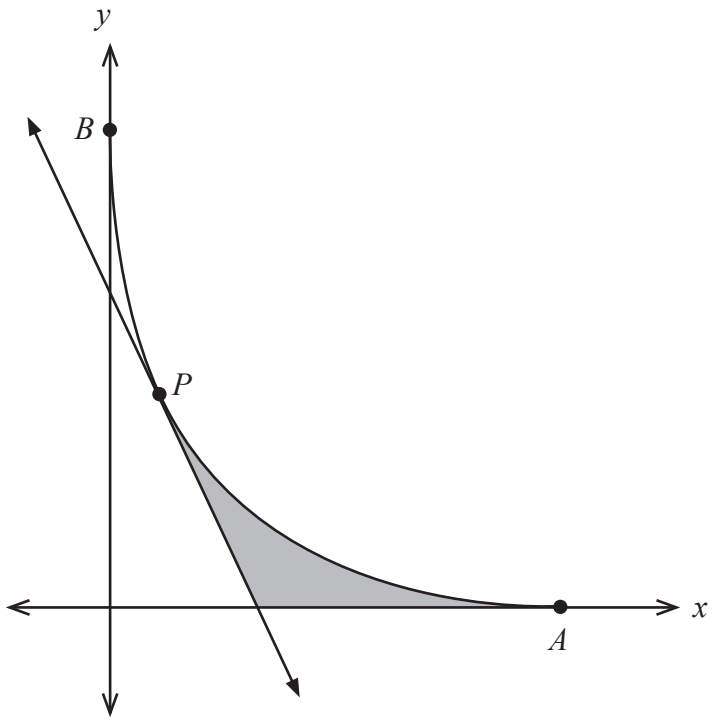
(c) Determine the equation for the line of force that passes through the point $A\ (0,1)$.

(4 marks)

See next page

Question 12 **(8 marks)**

The diagram shows the curve with equation $\sqrt{x} + \sqrt{y} = 3$ where points A, B are the intercepts of this curve. A tangent is drawn to the curve at point $P(1,4)$.



(a) Show that the equation of the tangent is $2x + y = 6$. (3 marks)

The shaded region on the diagram is bounded by the curve, the tangent and the x axis.

(b) Determine the exact area of the shaded region. (5 marks)

See next page

Question 13 **(13 marks)**

A cable in a bridge is required to support a weight of 10 000 Newtons. Tina tests a random sample of 100 cables from a supplier. The sample mean is found to be 10 300 Newtons and the sample standard deviation 400 Newtons.

(a) Based on Tina's sample, obtain a 95% confidence interval for μ, the population mean cable strength. (4 marks)

(b) State whether each of the following statements is true or false. Provide reasons for your answer and state any assumptions.

(i) If another sample of 100 cables is taken, then the sample mean will fall within the confidence interval found at part (a). (2 marks)

(ii) If a single cable is selected at random, then the strength of the cable will fall within the confidence interval found at part (a). (2 marks)

See next page

Jon, a colleague of Tina, said, 'The cable strengths are not normally distributed, so the calculation for the confidence interval is incorrect'.

(c) How should Tina respond to Jon's comment? (2 marks)

A different sample of 36 cables is taken and it is found that the standard deviation is 500 Newtons. A confidence interval for the population mean cable strength is determined to be $9900 \leq \mu \leq 10\,200$.

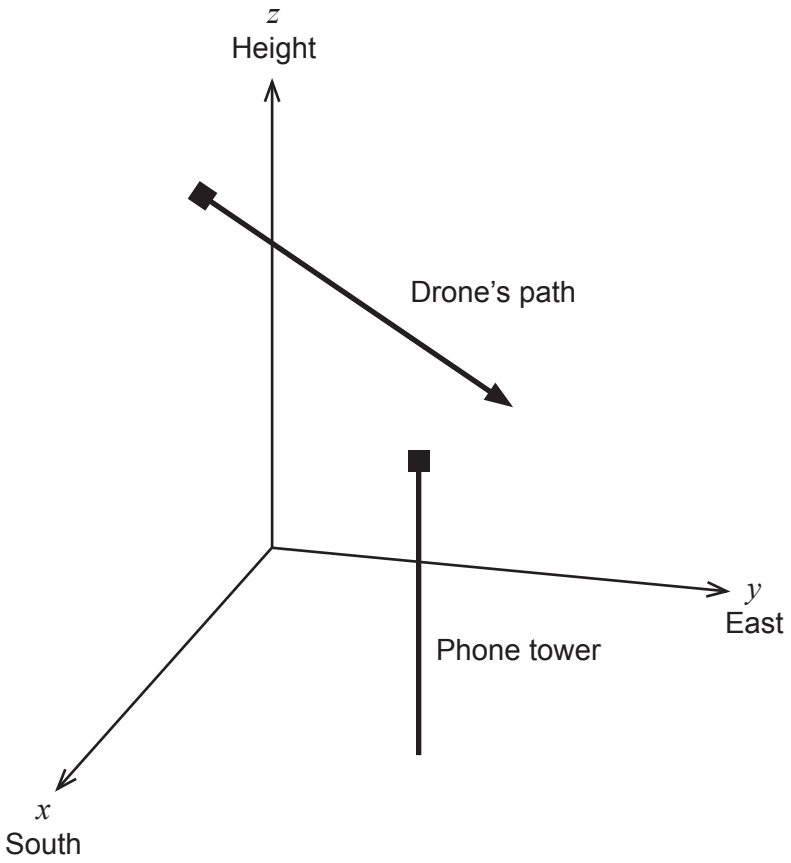
(d) Determine the confidence level, to the nearest 0.1%, used to calculate this interval. (3 marks)

Question 14 **(7 marks)**

A small drone is launched and, after hovering in an initial position, it flies in a straight line under the control of its operator. The position of the drone from the operator is given by

$\underset{\sim}{r}(t) = \begin{pmatrix} 100 + 0.5t \\ 0.6t \\ 50 - 0.02t \end{pmatrix}$ metres, where t is the time in seconds it has been flying in a straight line.

The top of a mobile phone tower is positioned at $200\underset{\sim}{i} + 150\underset{\sim}{j} + 30\underset{\sim}{k}$ relative to the operator i.e. the mobile phone tower is 30 metres tall.



(a) After two minutes of flight, how high is the drone above the ground? (2 marks)

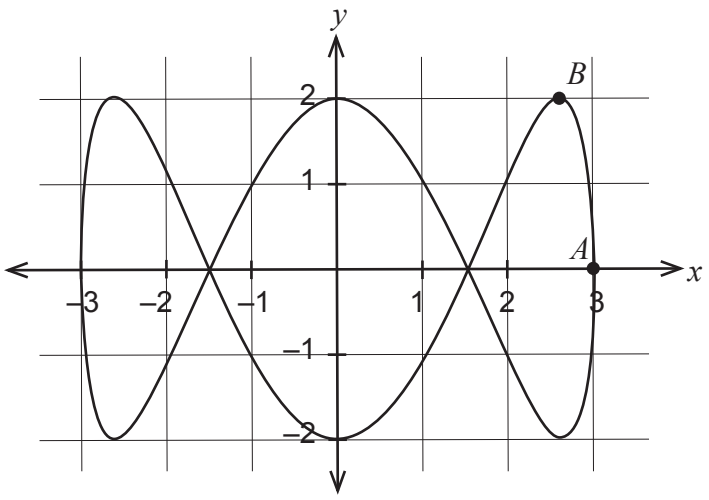
(b) Write the expression for the position vector of the drone from the top of the phone tower after t seconds. (1 mark)

The operator knows that the drone will not strike the mobile phone tower. However, the operator does not know that the drone will cause interference when it is less than 50 metres from the top of the tower.

(c) Determine whether the drone will cause interference to the mobile phone tower and, if so, for how long will this occur, correct to the nearest second. (4 marks)

Question 15 **(13 marks)**

A battery-powered model race car moves around a race track as indicated in the diagram below. The car's initial position is point A.



At any time t seconds, the velocity vector $\underset{\sim}{v}(t)$ of the model race car is given by:

$$\underset{\sim}{v}(t) = \begin{pmatrix} -\sin\left(\frac{t}{3}\right) \\ 2\cos(t) \end{pmatrix} \text{ metres per second.}$$

(a) Determine the initial velocity vector and show this on the diagram above. (2 marks)

(b) Write an expression that will determine the change in displacement over the first $\frac{3\pi}{2}$ seconds. (2 marks)

See next page

(c) Determine the displacement vector $\underset{\sim}{r}(t)$. (3 marks)

It can be shown that the model race car's speed is at a minimum when it reaches point B on the track, one of the sharpest points on the curve.

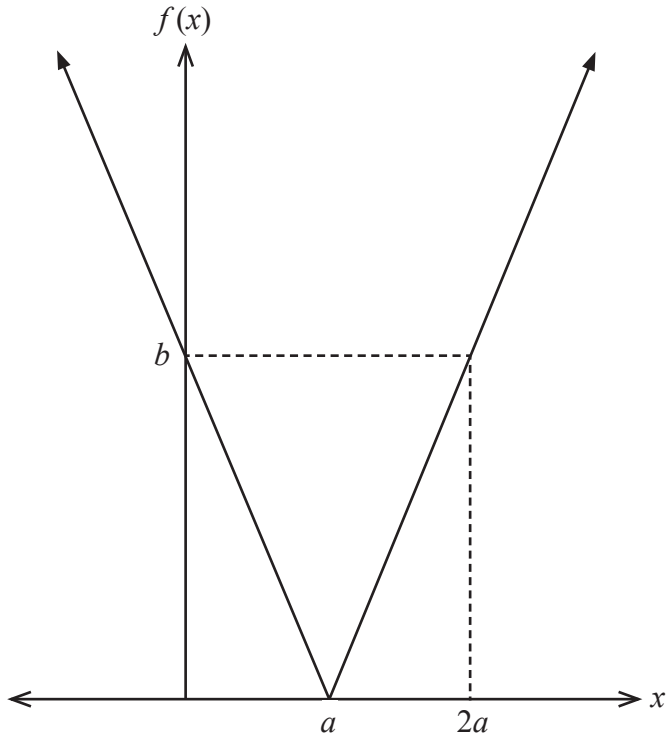
(d) Determine the acceleration vector $\underset{\sim}{a}$ when the car reaches point B, giving components correct to 0.01. (3 marks)

(e) Determine the distance, correct to 0.01 metres, that the model race car travels in completing one lap of the track. (3 marks)

See next page

Question 16 **(8 marks)**

Function f is defined by its graph shown below. The constants $a,\ b > 0$ where $b > a$.



(a) Determine the defining rule for function $f(x)$ in terms of $a,\ b$. (3 marks)

(b) By using the substitution $u = 2x - a$, determine an expression, in terms of a, b, for the value of $\int_{\frac{a}{2}}^{a} f(2x - a)\,dx$. (5 marks)

See next page

Question 17 **(11 marks)**

After t seconds, the displacement x centimetres of a small mass attached to a spring, oscillates about a fixed point O according to the differential equation $\frac{d^2x}{dt^2} = -\pi^2 x$.

The initial velocity is 8π centimetres per second and the initial displacement is zero.

(a) Determine the function $x(t)$ that gives the displacement of the mass at time t. (3 marks)

(b) Calculate the distance the mass travels during the first 5 seconds. (3 marks)

See next page

The differential equation $\frac{d^2x}{dt^2} = -\pi^2 x$ assumes that the amplitude of oscillation A is a constant over time.

Now assume that friction reduces the amplitude of the oscillation according to the equation $\frac{dA}{dt} = -0.4A$. Also assume $A(0) = 8$ centimetres.

(c) Determine the function $A(t)$ that gives the amplitude of the mass. (2 marks)

As time passes, the amplitude continues to decrease to the point at which the small mass appears to stop oscillating. This occurs when the amplitude is less than 0.01 cm.

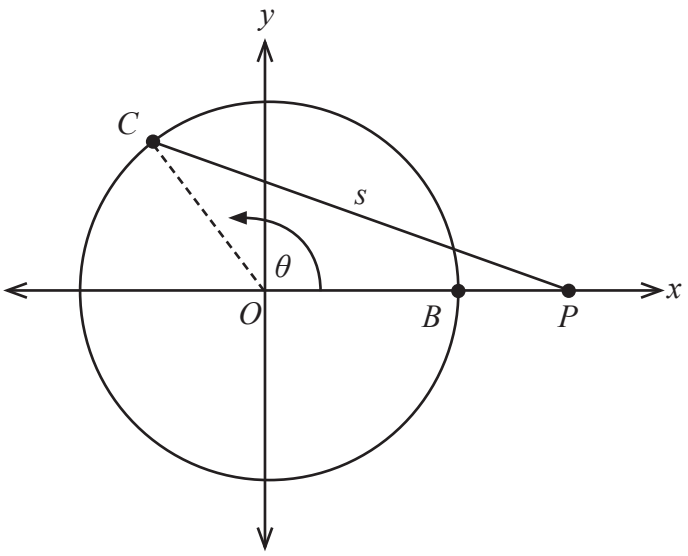
(d) Determine, correct to the nearest 0.1 seconds, how long it takes for the small mass to appear to stop oscillating. (3 marks)

See next page

Question 18 **(10 marks)**

A young child rides on a merry-go-round at a carnival. The merry-go-round has a radius of 5 metres and completes one revolution every 12 seconds. The parent of the young child stands and watches at point P, exactly 3 metres away from point B.

The ride begins at point B, when the child is closest to the parent, and the merry-go-round rotates in an anti-clockwise direction at a constant speed. At any point in time, point C is the position of the child on the merry-go-round.



Let t = the number of seconds the ride has been in progress (from starting at point B)
$s = PC$ = the distance that the child is from the parent (metres)
θ = size of $\angle BOC$ (radians)

(a) Show that $\frac{d\theta}{dt} = \frac{\pi}{6}$ radians per second. (1 mark)

(b) Show that $s^2 = 89 - 80\cos\theta$. (1 mark)

See next page

(c) By differentiating $s^2 = 89 - 80\cos\theta$ implicitly with respect to time t, determine, correct to the nearest 0.01 metre per second, the rate at which the child is moving away from the parent when the ride has been in progress for 4 seconds. (4 marks)

The parent notices that the child appears to move away from point P at varying speeds.

(d) Determine the value for $\cos\theta$ when the rate $\frac{ds}{dt}$ is a maximum. (4 marks)

See next page

Question 19 **(7 marks)**

Consider the complex equation $2z^6 = 1 + \sqrt{3}i$.

(a) Solve the above equation, giving solutions in polar form $rcis\theta$ where $0 < \theta < \frac{\pi}{2}$. (4 marks)

See next page

Now consider the equation $2z^n = 1 + \sqrt{3}i$, where n is a positive integer.

(b) If $2z^n = 1 + \sqrt{3}i$ has roots so that there are exactly 3 roots (and only 3) that lie within the first quadrant of the complex plane, determine the possible value(s) of n. Justify your answer. (3 marks)

End of questions

Supplementary page

Question number: _____

Supplementary page

Question number: _____

Supplementary page

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Supplementary page

Question number: _____

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